

# On the Scaling Function of Lyapunov Exponents for Intermittent Maps

R. Stoop

Institute of Mathematics, Swiss Federal Institute of Technology (ETH), CH-8092 Zurich, Switzerland

J. Parisi

Physical Institute, University of Bayreuth, W-8580 Bayreuth, Germany

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The scaling function of Lyapunov exponents for intermittent systems is full of particularities if compared with hyperbolic cases or the usual, nonhyperbolic, parabola. One particularity arises when this function is calculated from finite-time Lyapunov exponents: Different scaling properties with respect to the length of the finite-time chains emerge. As expected from random walk models, the scaling of an ensemble with non-Gaussian fluctuations evolves for certain values of the external parameter.

The evaluation of fractal dimensions and Lyapunov exponents has obtained wide-spread interest, especially, since it was discovered that not only for models, but also for experimental data these concepts can be applied to characterize the statistical behavior of a system. A particularly difficult thing is the evaluation of the scaling function of Lyapunov exponents,  $\phi(\lambda)$  [1], for intermittent systems (if compared with the hyperbolic or the logistic cases). In this communication, we show scaling properties which depend on the length (i.e., the number of terms) of the finite-time Lyapunov exponents used for the evaluation of this function. We compare the hyperbolic case with different kinds of intermittent maps and find qualitatively different behavior.

As the length of the finite-time Lyapunov exponents increases, the average Lyapunov exponent converges, a.e., towards the Lyapunov exponent of the system;  $\phi(\lambda)$ , which describes the fluctuation of these values around the true exponent of the system, must, in this sense, be regarded as an entropy of the large-deviation property [2]. In Fig. 1, we show  $\phi(\lambda)$  for a hyperbolic system; the reduced map of a supercritical tent map has been considered (if not the reduced map is taken, a strange repeller is obtained). The scaling property of the obtainable width of  $\phi(\lambda)$  with respect to the length

$n$  is indicated by the inset in the picture; as expected from theory, it scales as  $d(n) \sim n^{-1/2}$ .

A completely different behavior is found for Manneville's intermittent map in the version of Zumofen and Klafter [3]. Depending on the exponent  $z$  of the map, the results in Figs. 2 and 3 are obtained. In

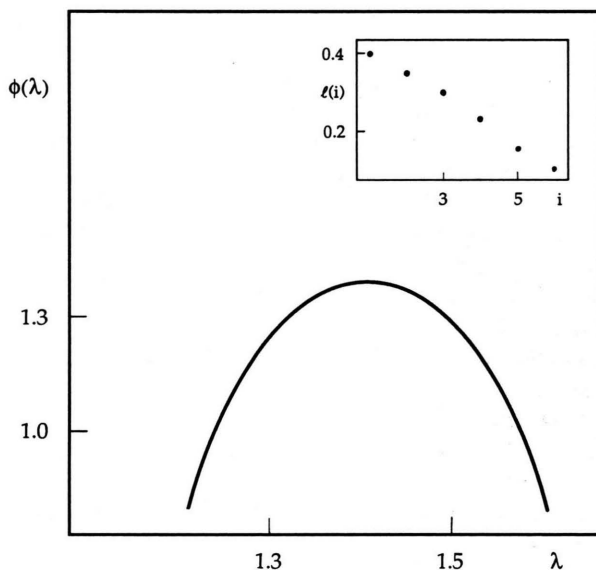


Fig. 1. Scaling function  $\phi(\lambda)$  of Lyapunov exponents of the reduced tent map, evaluated from 2000 chains of length  $n = 5$ . The scaling properties of the numerically obtained part of  $\phi(\lambda)$  is indicated in the inset. There, we display the width of this region versus the number of times the original length of the chain has been doubled.

Reprint requests to Prof. Dr. J. Parisi, Physical Institute, University of Bayreuth, P.O. Box 10 12 51, W-8580 Bayreuth, Germany.

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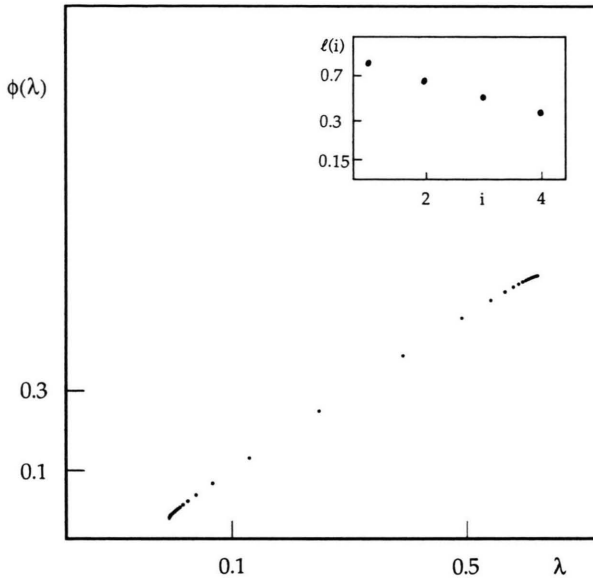


Fig. 2. Scaling function  $\phi(\lambda)$  of Lyapunov exponents of the intermittent map for  $z = 4$ , evaluated from 2000 chains of length  $n = 250$ . Again, the scaling behavior as a function of  $n$  is indicated in the inset. Note the logarithmic scale used for the ordinate. The result is characteristic for  $z > 2$ .

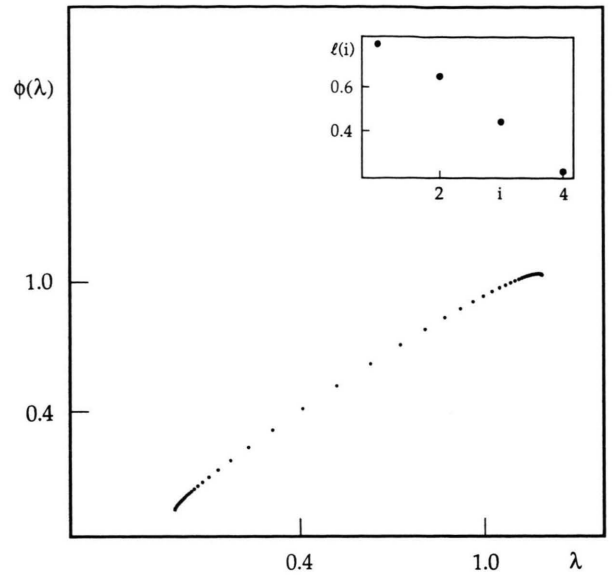


Fig. 3. Scaling function  $\phi(\lambda)$  of Lyapunov exponents of the intermittent map for  $z = 4/3$ , evaluated from 2000 chains of length  $n = 125$ . Again, the scaling behavior as a function of  $n$  is indicated in the inset. Note the linear scale used for the ordinate. The result is characteristic for  $1 < z < 3/2$ .

Fig. 2, for  $z=4$ , the fluctuations are of Lévy type (as shown in [4]), the Lévy exponent  $\alpha$  belonging to the class  $1 < \alpha < 2$ . For  $3/2 < z < 2$ , the fluctuations are also of Lévy type, with  $0 < \alpha < 1$ , whereas for  $1 < z < 3/2$ , the fluctuations are Gaussian (cf. Figure 3). The l.h.s.'s tendency to evade a calculation is intimately connected with the intermittency exponent  $z$ . The scaling property of the numerically obtainable interval of  $\phi(\lambda)$  is documented in the insets of Figures 1–3. In Figs. 1 and 3, a linearly scaled ordinate is shown; the scale in Fig. 2 is logarithmic. Note that in the range  $3/2 < z < 2$  almost no variation is observed for the width of the obtainable interval. This fact is in accordance with the existence of an invariant measure associated with these values of  $z$  [5]. The r.h.s. of the scaling function cannot be evaluated with the present approach. However, the generalized (bivariate) thermodynamic partition function [6, 7] would enable to find also this branch.

Let us finally remind that these scaling properties are intimately connected with the scaling properties of the leftmost interval, if a generating partition is used for this system. The connection between the power-law scaling of the length of this interval and the power-law scaling of the measure associated with it is straightforward but nontrivial and, for high  $z$ -values, not easy to observe numerically. Let us further note that the same behavior can be used to explain the anomalous diffusion property shared by intermittent maps [5]. For a recent review of these properties, derived via Laplacian transforms and renewal theory, we refer to [3].

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